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How to Measure the Magnetic Moment of the Tau Lepton

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We present a method for obtaining bounds on the magnetic moment of the τ lepton. In order to do this, we study the radiative decay $W \to \tau\nu\gamma$ as a function of the anomalous magnetic moment of the τ, a_τ. One can obtain bounds as good as $a_\tau < 4.05 \times 10^{-2}$, 2.25×10^{-2}, 4.5×10^{-3}, and 2.5×10^{-3} at the present Fermilab, future Fermilab, SSC, and LHC, respectively.

The anomalous magnetic moments of the electron (Kinoshita, 1990*a*) and the muon (Kinoshita and Marciano, 1990; Kinoshita, 1990*b*; Samuel and Li, 1991) provide very precise tests of quantum electrodynamics (QED). In the case of the electron we have an almost pure QED system. However, in the case of the muon, the hadronic contribution is large and the weak-interaction contribution will likely be measured in the new $g-2$ muon experiment, which is underway at Brookhaven National Laboratory. This will provide a good test of the Standard Model (SM), assuming a_μ (hadronic) can be determined more precisely.

The τ lepton was discovered in 1975 (Perl *et al.*, 1975). All evidence indicates that it is a SM lepton (Perl, 1990; Barish and Stroynowski, 1988). Recently the magnetic moment μ of the τ was calculated (Samuel *et al.*, 1991, 1992). The result is

$$a_\tau = \left(\frac{g-2}{2}\right) = 11773(3) \times 10^{-7} \tag{1}$$

where

$$\mu = \frac{ge\hbar}{2m_\tau c} \tag{2}$$

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An earlier attempt (Narison, 1978) to calculate a_τ is unfortunately wrong. A rough estimate, in agreement with our result, was made by Barish and Stroynowski (1988).

We now turn to the measurement of a_τ. A bound from $Z^0 \to \tau^+\tau^-\gamma$ has recently been obtained (Grifols and Mendez, 1991). It is

$$|a_\tau| < 0.11 \tag{3}$$

Several ways of measuring a_τ have been proposed. One such method is to observe τ pair production in heavy-ion collisions (del Aguila *et al.*, 1991). Also, one may use the radiation amplitude zero in radiative τ decay (Samuel *et al.*, 1991, 1992), $\tau \to e\nu\bar{\nu}\gamma$. Another possibility is to measure a_τ by channeling in a bent crystal (Baryshevskii, 1979; Lyuboshits, 1980; Pondrom, 1982; Kim, 1983; Kim and Sun, 1984; Sun, 1985, 1987; Chen *et al.*, 1992).

Here we propose another method which can be used to measure a_τ. It is to observe the radiative W decay into τ's,

$$W^\pm(P) \to \tau^\pm(p_1) + \nu(p_2) + \gamma(k) \tag{4}$$

This experiment could be done at the Fermilab Tevatron now or in the future, at the SSC, or at the LHC. Here we consider the decay in (4) as a function of a_τ. Our result for the differential decay rate is (to simplify the notation, we now use $a = a_\tau$)

$$\frac{1}{\Gamma_0}\frac{d^2\Gamma}{dx\,dy} = \frac{\alpha}{2\pi}\Omega \tag{5}$$

where

$$\begin{aligned}
\Omega = {} & Z^2\frac{1 - x - (x^2/4)(1+y^2)}{x(1-y^2)} + \frac{Q_1^2 R a^2}{64}[8(1-x) + x^2(1-y^2)]x \\
& - \frac{aQZ}{4(1-y^2)}[-(x + xy^2 - 1)Q_1 - (2x-1)Q] \\
& + \frac{Q^2Z^2}{2Rx(1-y^2)^2}[2y^2 - 10 + x(x+2)(1-y^2)] \\
& - \frac{Q^2a^2}{8}x + 2\frac{Q^2Z^2 + aQQ_1RZ}{2R^2}\frac{x}{1-y^2} - \left(\frac{K-1}{4}\right)Zxy \\
& + \frac{(K-1)^2}{16}\left[1 - x + \frac{(1-y^2)(1+x)}{2}\right]x - \frac{Q^2(K-1)^2}{8R}x(1+x) \\
& + \frac{Q^2(K-1)Z}{R}\frac{xy}{1-y^2} - a\frac{Q(K-1)}{2} \\
& \times \left[\frac{Q - yQ_1}{R(1-y^2)} + \frac{y(2+x)Q_1 - (6-3x)Q}{8}\right]x
\end{aligned} \tag{6}$$

where $x = 2(p_1 + p_2)k/M_W^2$ and

$$y = \frac{(p_1 - p_2)k}{(p_1 + p_2)k} \tag{7}$$

The zero factor is $Z = (y - 1)$ and

$$\Gamma_0 = \frac{\alpha M_W}{12 \sin^2 \theta_W} = \Gamma_0(W^+ \to \tau^+ \nu) \tag{8}$$

$R = (M_W/m_\tau)^2$ and the magnetic moment of the W is given by

$$\mu = \frac{e}{2M_W}(K + 1) \tag{9}$$

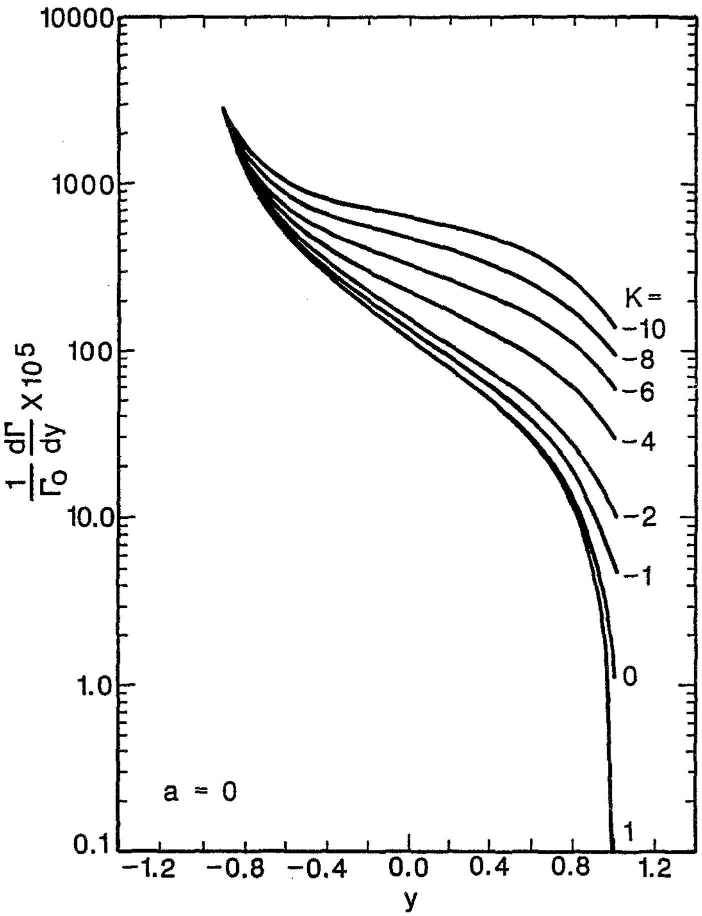

Fig. 1. Plot of $(1/\Gamma_0)\, d\Gamma/dy$ versus y for $a = 0$ and various values of K. The RAZ at $y = 1$ occurs for $K = 1$ $(0.2 \leq x \leq 1.0)$.

where $K = 1$ for the SM, M_W is the W mass, $\alpha = 1/128$, $\sin^2\theta_W = 0.233$, and $x = 2E\gamma/M_W$, in the W center-of-mass frame.

The radiation amplitude zero (RAZ) occurs at the edge of phase space $Z = 0$ or $y = 1$. Here Q is the charge of the W and Q_1 is the charge of the τ. Note that the RAZ occurs at $y = 1$ for *both* W^+ and W^- radiative decay.

We now integrate over x $(0.2 \le x \le 1.0)$. The results are shown in Figs. 1 and 2. Figure 1 gives $d\Gamma/dy$ for $a = 0$ and various values of K and Fig. 2 gives $d\Gamma/dy$ for $K = 1$ and various values of a. The RAZ is clearly visible at $y = 1$. It can be seen that the RAZ occurs only if $a = 0$ *and* $K = 1$.

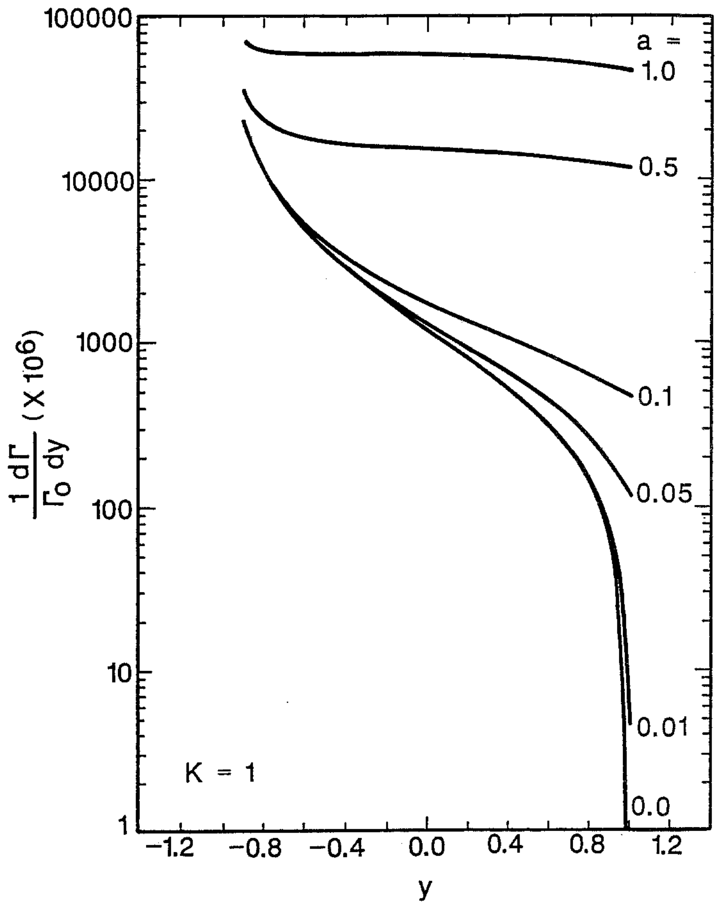

Fig. 2. Plot of $(1/\Gamma_0)\, d\Gamma/dy$ versus y for $K = 1$ and various values of a. The RAZ at $y = 1$ occurs for $a = 0$ $(0.2 \le x \le 1)$.

We now integrate over y $(0 \leq y \leq 1)$ as well as x $(0.4 \leq x \leq 1)$, to obtain the branching ratio B. The results are shown in Fig. 3, where we have plotted B versus a for $K = 1$ and B versus K for $a = 0$. It can be seen that the minimum for B occurs when both $K = 1$ *and* $a = 0$. We can improve the sensitivity to a by choosing a smaller range for x, $x_0 \leq x \leq 1$,

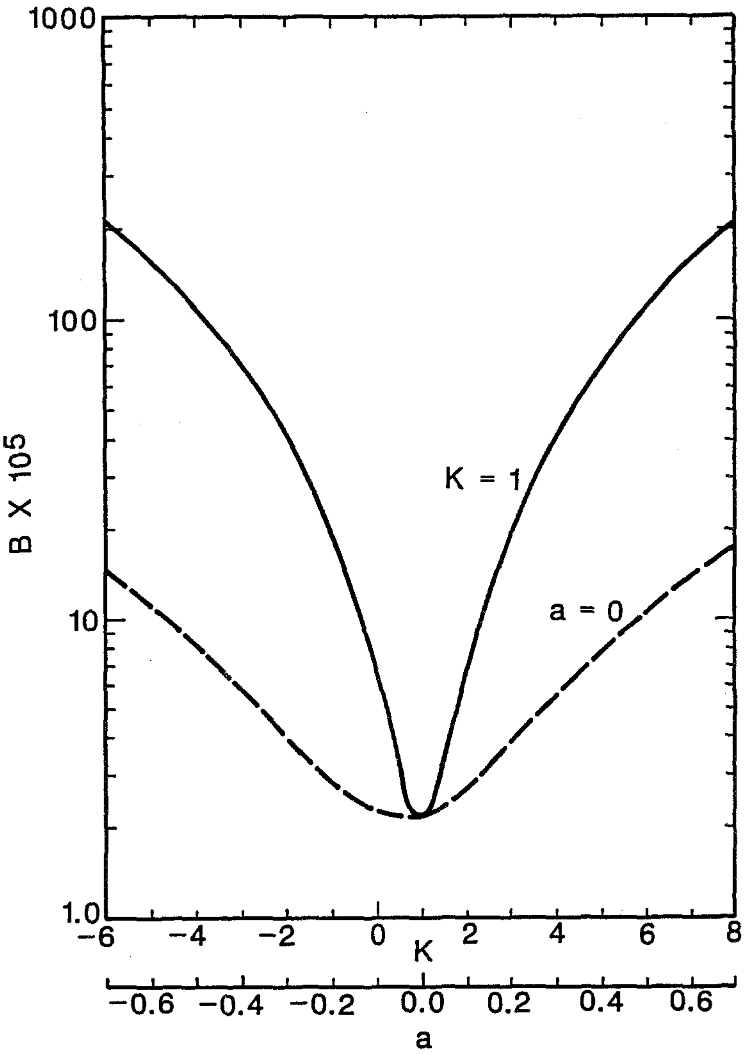

Fig. 3. Plots of B versus a for $K = 1$ and B versus K for $a = 0$. $(0.4 \leq x \leq 1$ and $0 \leq y \leq 1.0)$. The minimum of B occurs when *both* $a = 0$ and $K = 1$.

with $x_0 > 0.4$, but then, of course, B decreases. Note that B is much more sensitive to a for $K=1$ than to K for $a=0$.

We now compute the number of events one will be able to obtain at the present (future) Fermilab Tevatron, the SSC, and the LHC at CERN after 1 year of running. We use $\int \mathscr{L}\, dt = 100$ (1000) pb^{-1} for the present (future) Fermilab. We allow a factor of 3 for experimental acceptances and efficiencies. The number of $W \to \tau\nu$ events at the present (future) Fermilab Tevatron, the SSC, and the LHC is 7×10^4 (7×10^5), 2.1×10^8, and 1.8×10^9, respectively. In addition there will be a factor of 2 if one includes both W^+ and W^-. From the number of events we then calculate the bounds on a one can obtain at the 90% confidence level. The results are shown in Table I. The ranges are $x_0 \leq x \leq 1$ and $y_0 \leq y \leq 1$. It can be seen that the best bound is $a < 2.5 \times 10^{-3}$ for the LHC. At the SSC the best bound is $a < 4.5 \times 10^{-3}$ and for the present (future) Fermilab Tevatron it is $a < 4.1 \times 10^{-2}$ (2.3×10^{-2}). These bounds all occur for $x_0 = 0.4$ and $y_0 = 0$.

In conclusion we have proposed a new method to measure the anomalous magnetic moment of the tau (τ) lepton at hadron colliders. We have shown that one can obtain very good bounds: $a < 4.05 \times 10^{-2}$ (2.25×10^{-2}), 4.5×10^{-3}, and 2.5×10^{-3} at the present (future) Fermilab, the SSC, and the LHC, respectively.

Table I. Bounds on a_τ [a]

x_0	y_0	a_1	a_2	a_3	a_4
0.4	0	4.1	2.3	0.5	0.3
0.6	0	4.4	2.4	0.5	0.3
0.6	−0.5	4.7	2.6	0.5	0.3
0.4	0.5	4.1	2.3	0.5	0.3
0.8	−0.5	5.8	3.2	0.6	0.3
0.8	0	5.5	3.0	0.6	0.3
0.8	0.5	5.9	3.3	0.7	0.4
0.5	0.6	4.5	2.5	0.6	0.3
0.2	0.7	4.3	2.4	0.6	0.3
0.2	0.5	4.3	2.4	0.6	0.3
0.4	−0.5	4.4	2.5	0.5	0.3
0.2	−0.5	4.7	2.6	0.6	0.3
0.2	0	4.3	2.4	0.5	0.3

[a] a_1 (a_2) is for the present (future) Fermilab Tevatron, a_3 is for the SSC, and a_4 is for the LHC and $a_i = a_\tau \times 10^2$, $i = 1, 2, 3, 4$.

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